

Topics 3: Excess carrier behavior in semiconductor devices

Virtually all semiconductor devices in active mode involve the generation, decay, or movement of carriers from one region to another

Carrier population (n , p) that is different from the population at rest (n_0 , p_0) is by definition excess carriers ($\Delta n = n - n_0$, $\Delta p = p - p_0$)

The excess carrier behavior determines how a device work

Introduction

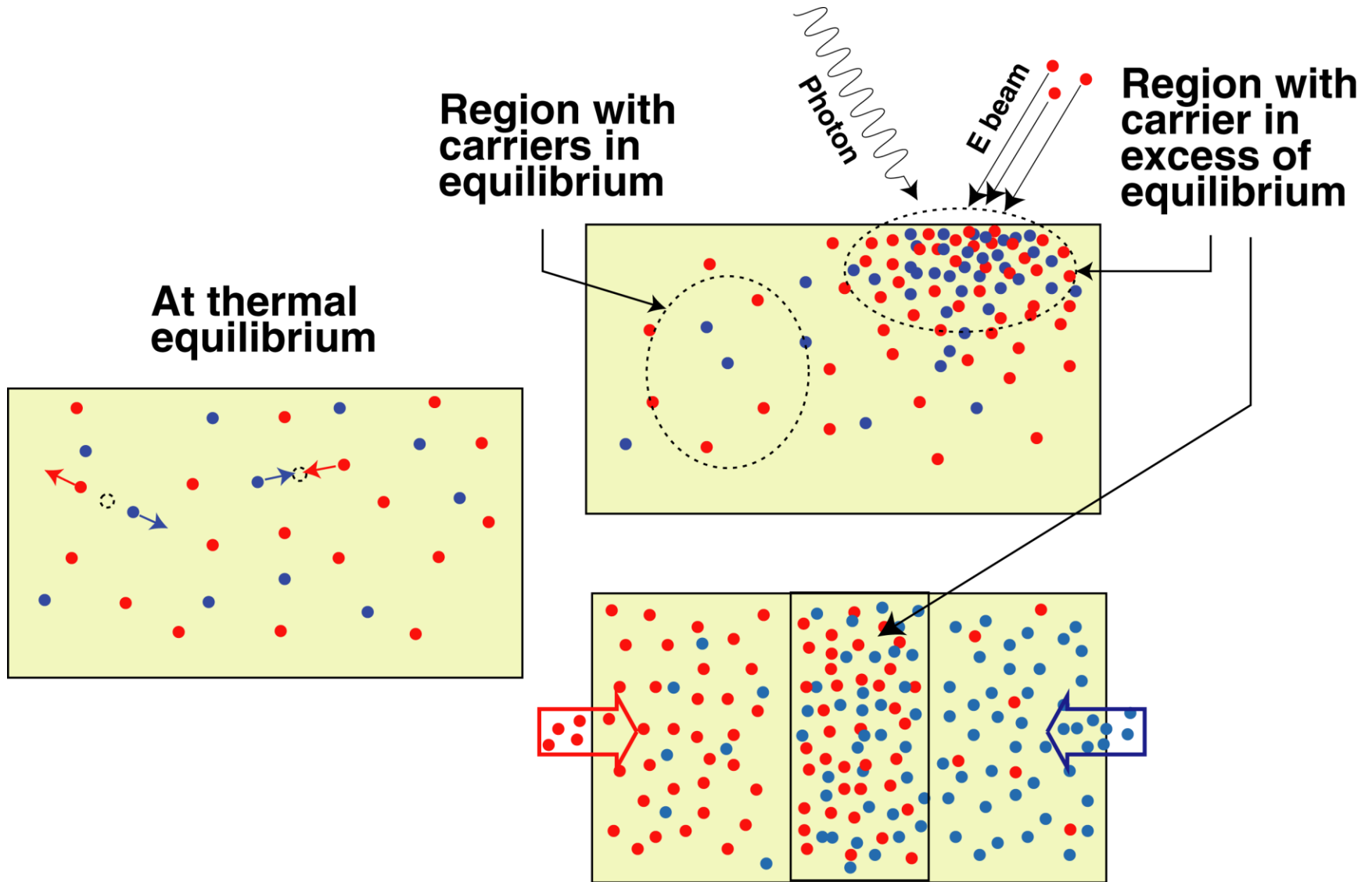
- *Devices in active states involve **non-equilibrium**, and/or **non-uniform**, and/or transient (**non-steady state**), and/or **non-ohmic carrier injection***
- Non-equilibrium carrier behavior
 - Distributions $n(x, t)$, $p(x, t)$
 - Relaxation and recombination
 - Migration: drift and diffusion
- Some device examples
 - Photodetectors, solar cells, LEDs and lasers
 - Cathodoluminescence (Field-emission displays FEDs)

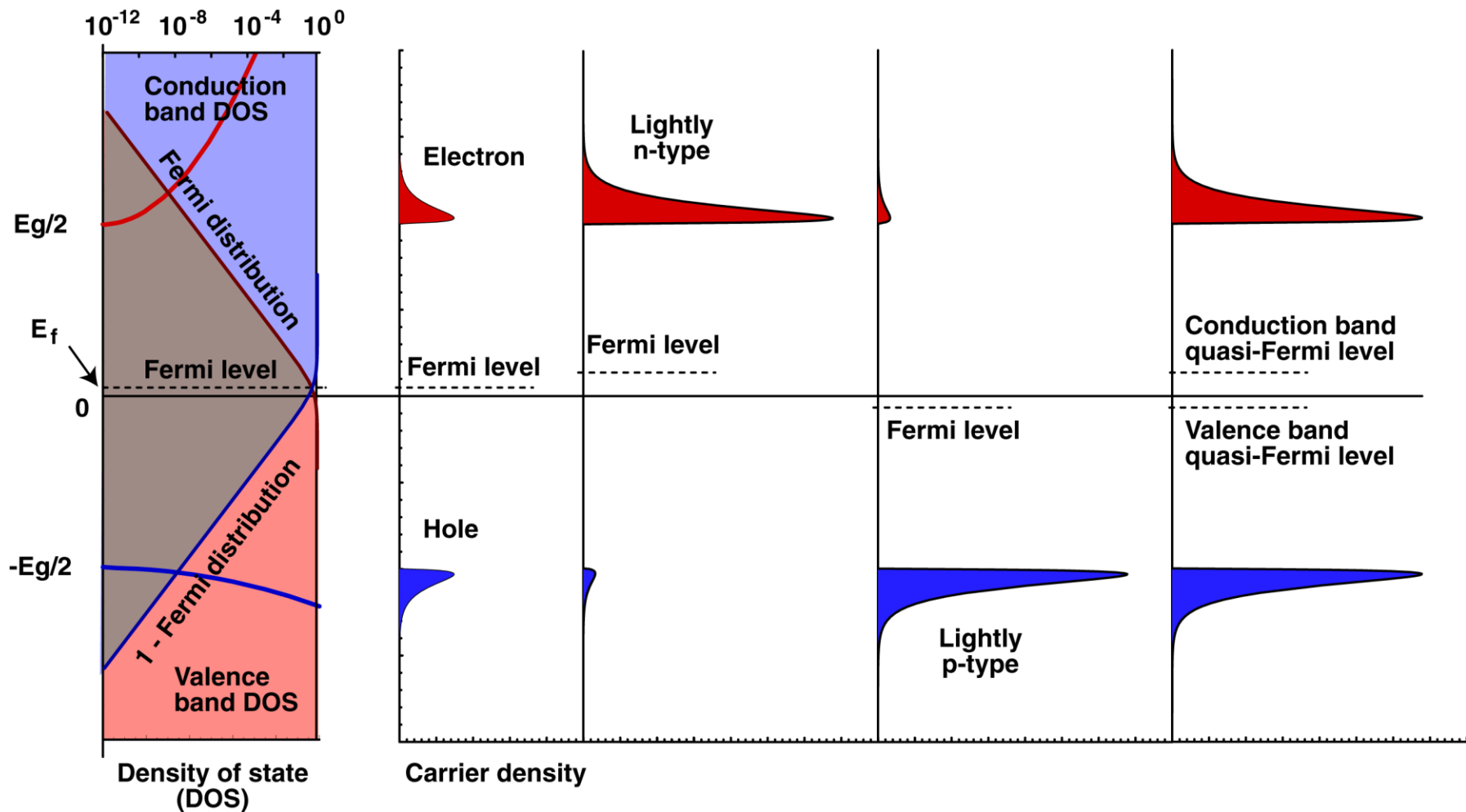
Non-equilibrium carrier behavior

Some Key Concepts

- Excess carriers: quasi-Fermi levels**
- Carrier injection: optical injection (optical pumping, absorption), e-beam injection (electron pumping)**
- Carrier relaxation, recombination, lifetime: Ionization or avalanche. Radiative or non-radiative recombination, direct recombination or trap-mediated recombination.**
- Carrier transport: drift and diffusion**

Non-equilibrium carrier behaviors





Density of state (linear scale) and Fermi distribution (log scale)

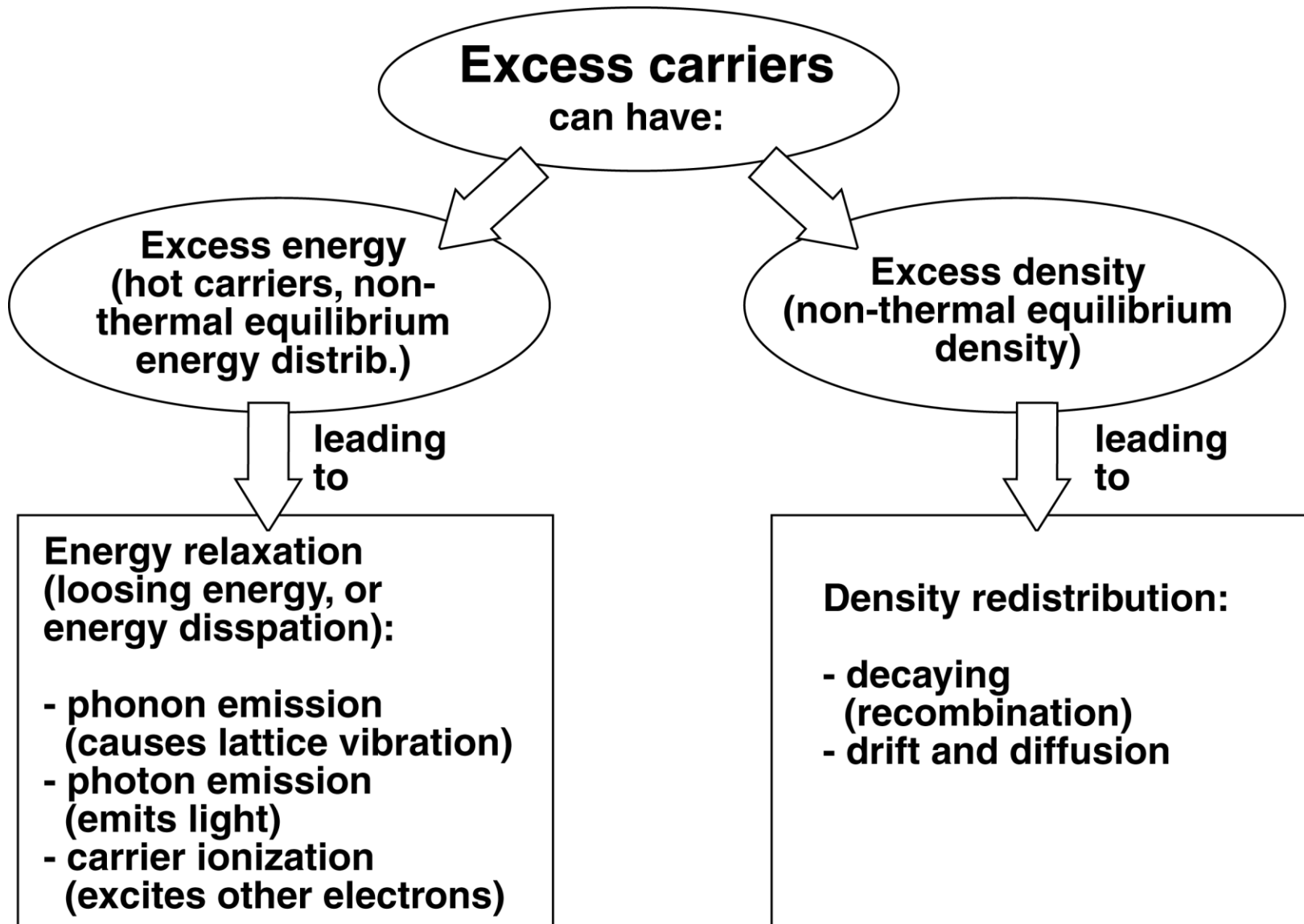
Intrinsic carrier density

Slightly n-type carrier density

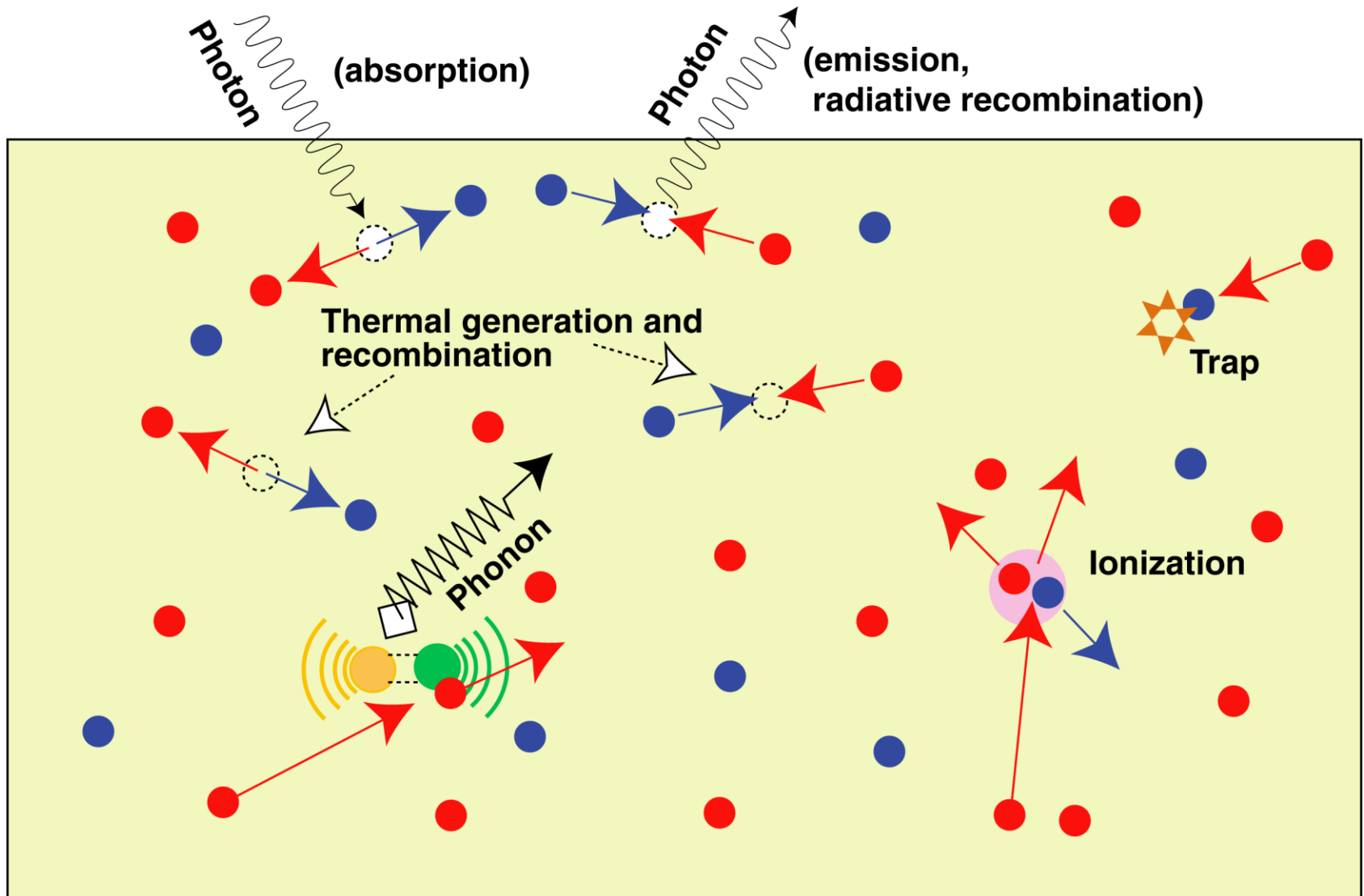
Slightly p-type carrier density

Both n and p, not in thermal equilibrium

Non-equilibrium carrier behaviors



Excitation and Relaxation



Semiconductors in Nonequilibrium Conditions

Injection $\Rightarrow$ A process of introducing excess carriers in semiconductors.

Generation and recombination are two types:

- (i) **Direct band-to-band** generation (G) and recombination (R) and
- (ii) the recombination through allowed energy states within the bandgap, referred to as traps or recombination centers.

Direct band-to-band generation and recombination

Thermal equilibrium:

The random generation-recombination of electrons-holes occur continuously due to the thermal excitation.

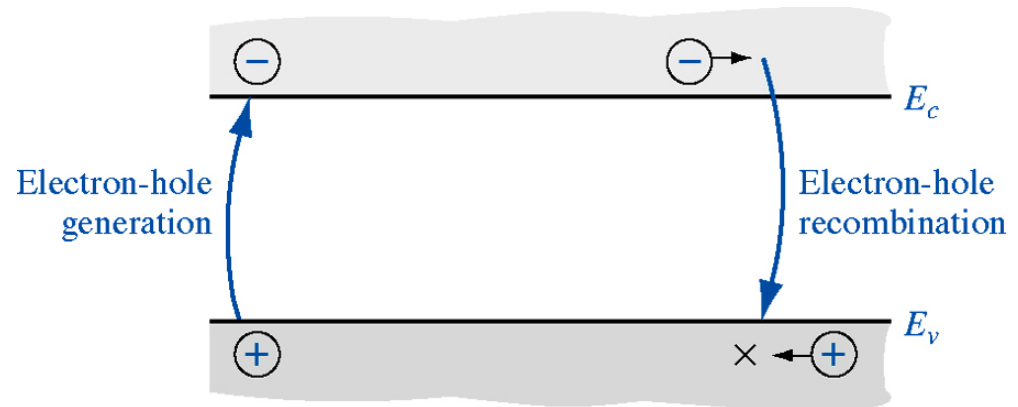
In direct band-to-band generation-recombination, the electrons and holes are created-annihilated in pairs:

$$G_{n0} = G_{p0}$$

$$R_{n0} = R_{p0}$$

At thermal equilibrium, the concentrations of electrons and holes are independent of time; therefore, the generation and recombination rates are equal, so we have,

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$



Indirect recombination:

In real semiconductors, there are some crystal defects and these defects create discrete electronic energy states within the forbidden energy band.

Recombination through the defect (trap) states is called indirect recombination

The carrier lifetime due to the recombination through the defect energy state is determined by the **Shockley-Read-Hall** theory of recombination.

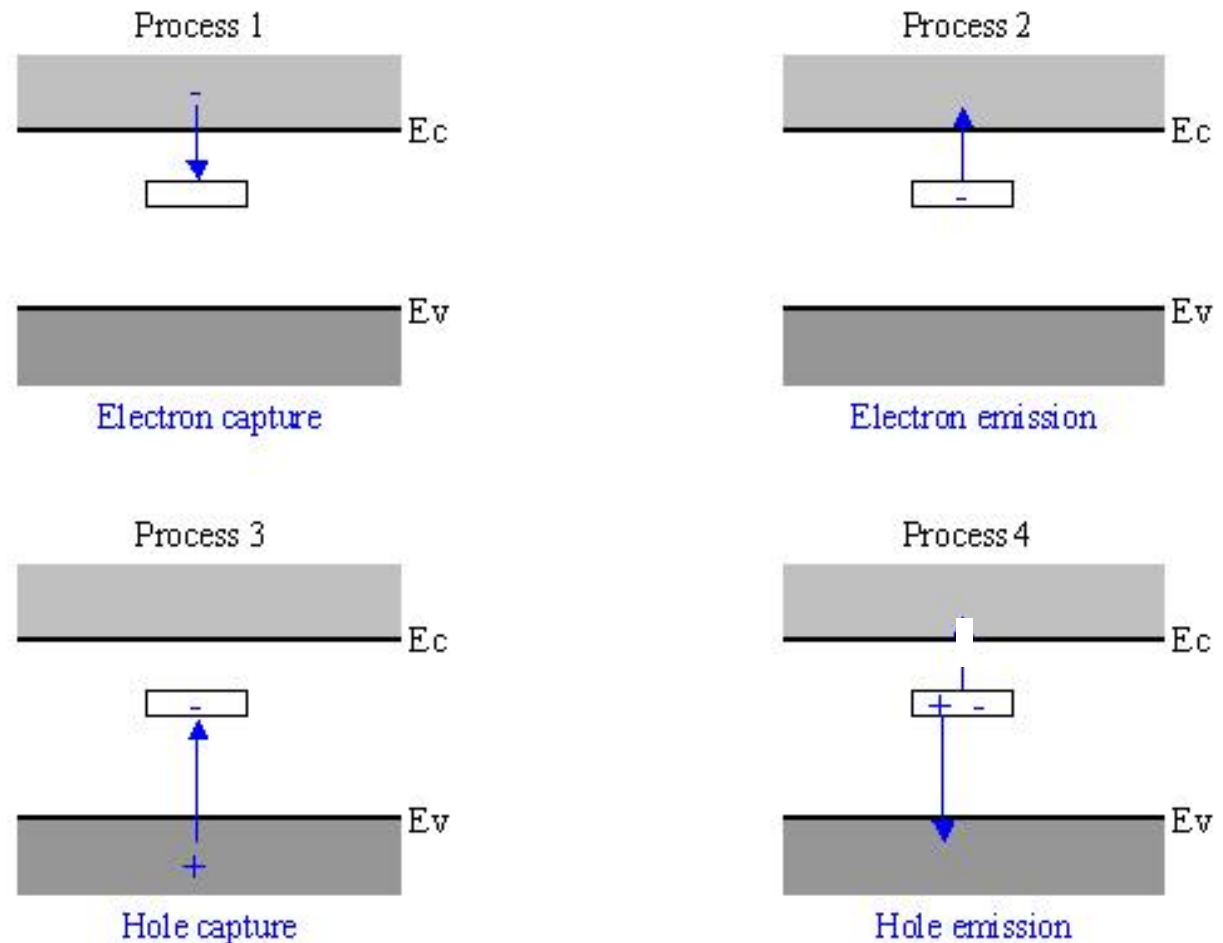
Shockley-Read-Hall recombination:

Shockley-Read-Hall theory of recombination assumes that a single trap center exists at an energy E_t within the bandgap.

Four basic trap-assisted recombination processes

The trap center here is **acceptor-like**.

It is **negatively charged** when it contains an electron and is **neutral** when it does not contain an electron.



The four basic trapping and emission processes for the case of an acceptor-type trap.

Excess carrier generation and recombination

<u>Symbol</u>	<u>Definition</u>
n_0, p_0	Thermal equilibrium electron and hole concentrations (independent of time)
n, p	Total electron and hole concentrations (may be functions of time and/or position).
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position).
G_{n0}, G_{p0}	Thermal electron and hole generation rates ($\text{cm}^{-3}\text{s}^{-1}$)
R_{n0}, R_{p0}	Thermal equilibrium electron and hole recombination rates ($\text{cm}^{-3}\text{s}^{-1}$)
g'_n, g'_p	Excess electron and hole generation rates ($\text{cm}^{-3}\text{s}^{-1}$)
R'_n, R'_p	Excess electron and hole recombination rates ($\text{cm}^{-3}\text{s}^{-1}$)
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes (s)

Nonequilibrium Excess Carriers

When excess electrons and holes are created,

$$n(t) = n_0 + \delta n(t), \quad p(t) = p_0 + \delta p(t) \quad \text{and} \quad np \neq n_i^2$$

The excess electrons and holes are created in pairs,

$$g'_n = g'_p \quad \text{and} \quad \delta n(t) = \delta p(t)$$

The excess electrons and holes recombine in pairs,

$$R'_n = R'_p$$

The excess carrier will decay over time and the decay rate depends on the concentration of excess carrier.

$$\frac{dn(t)}{dt} \propto [n_i^2 - n(t)p(t)] \quad \text{or,} \quad \frac{dn(t)}{dt} = \alpha_r [n_i^2 - n(t)p(t)]$$

α_r is the constant of proportionality for recombination.

Nonequilibrium Excess Carriers

$$\frac{dn(t)}{dt} = \alpha_r [n_i^2 - n(t)p(t)]$$

$$n(t) = n_0 + \delta n(t), \quad p(t) = p_0 + \delta p(t) \quad \text{and} \quad \delta n(t) = \delta p(t)$$

$$\frac{d\delta n(t)}{dt} = -\alpha_r \delta n(t) [(n_0 + p_0) + \delta n(t)]$$

For low-level injection condition: $(p_0 + n_0) \gg \delta n(t)$

For p-type material, $p_0 \gg n_0$

$$\text{Thus,} \quad \frac{d\delta n(t)}{dt} = -\alpha_r \delta n(t) p_0$$

The solution for minority carrier is $\delta n(t) = \delta n(0)e^{-\alpha_r p_0 t} = \delta n(0)e^{-t/\tau_{n0}}$

where τ_{n0} is the excess minority carrier lifetime for electron and $\tau_{n0} = 1/(\alpha_r p_0)$

Excess carriers

We consider here absence of surface or bulk recombinations

Excess carrier concentration in E_{VB} and E_{CB} depends on:

- Carrier life time
- Absorption profile
- Temperature

Excess carriers

Intrinsic carrier concentration similar to Si

$$n_i = p_i = 10^{10} \text{ cm}^{-3}$$

For n-type doping with majority carriers concentration

$$n = 10^{16} \text{ cm}^{-3}$$

Mass action law:

$$p = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ cm}^{-3}$$

$$p = \frac{n_i^2}{n}$$

Optical excitation perturbs this relation

Minority carriers concentration $p = 10^4 \text{ cm}^{-3}$

Stationary excess carrier concentration

P- photon flux $10^{17} \text{ cm}^{-2} \text{ s}^{-1}$

for $h\nu = 2 \text{ eV}$ (red light)

AM 1.5 at 84.4 mW cm^{-2}

τ - carrier lifetime $1 \mu\text{s}$

X_α - absorption of photons 10^{-3} cm^{-1}

within a volume of $1 \text{ cm}^{-3} \times 10 \mu\text{m}$ depth

$$\Delta p = \Delta n = \frac{P \tau}{X_\alpha}$$

$$\Delta p = \frac{10^{17} \cdot 10^{-6}}{10^{-3}} \left[\frac{\text{s}^{-1} \text{s}}{\text{cm}^{-3}} \right]$$

$$\Delta p = 10^{14} \text{ cm}^{-3}$$

n_i - intrinsic carriers SC

$$n_i = 10^{10} \text{cm}^{-3}$$

n - electrons in doped SC in the dark

$$n = 10^{16} \text{cm}^{-3}$$

p - holes in doped SC in the dark

$$p = 10^4 \text{cm}^{-3}$$

Δn - electrons in doped SC created by illumination

$$\Delta n = 10^{14} \text{cm}^{-3}$$

Δp - holes in doped SC created by illumination

$$\Delta p = 10^{14} \text{cm}^{-3}$$

n^* - electrons in doped SC under illumination

$$n^* = n + \Delta n = 10^{16} + 10^{14}$$

$$n^* = 10^{16} + 10^{14}$$

p^* - holes in doped SC under illumination

$$p^* = p + \Delta p = 10^4 + 10^{14}$$

$$p^* = 10^4 + 10^{14}$$

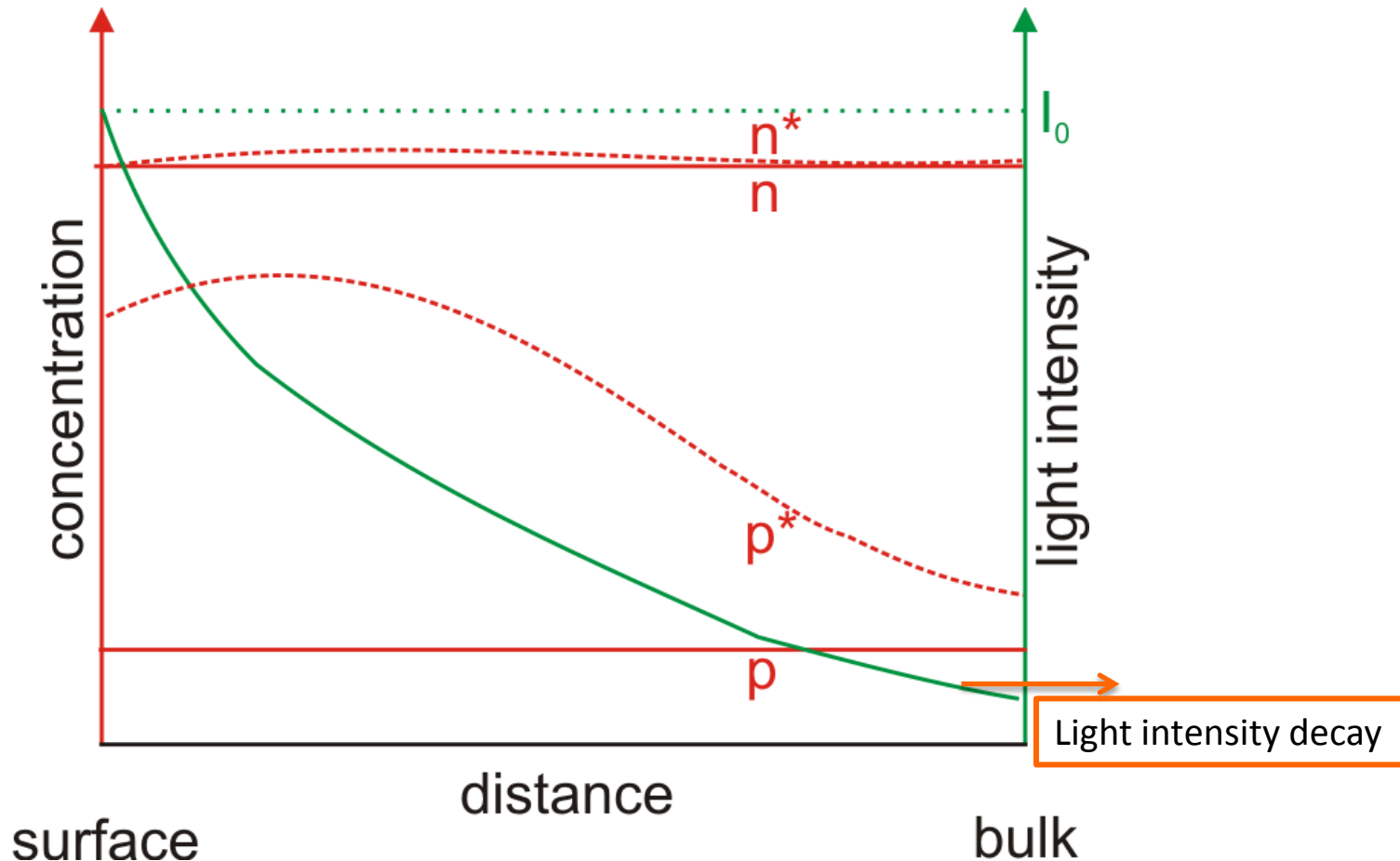
For **majority carriers** change by illumination is only **1%**

For **minority carriers** change by illumination is **drastical** – ten orders of magnitude

For n-type semiconductor:

- concentration of electrons coming from doping and thermal excitation is much higher than concentration of electrons coming from illumination
- concentration of holes coming from illumination is much higher than holes coming by thermal excitation

spatially dependent carrier concentration profiles in equilibrium (dark) and under illumination in comparison with the light absorption profile.



Whereas the excess majority carrier profile changes little (the change has been magnified in the figure), the excess minority carrier concentration p^* deviates strongly from the constant dark concentration (p).

Quasi Fermi levels, definitions

For stationary illumination and sufficiently long carrier life time, excess minority and majority carriers exist stationary at the respective band edges. Their excess carrier concentration relation defines a new quasi equilibrium and attempts have been made to describe this situation in analogy to the dark equilibrium terminology. Therefore one describes the Fermi level for an illuminated semiconductor in the framework of the equations derived for the non illuminated semiconductor. For n-type and p-type semiconductors, E_F was given by

$$E_F = E_{CB} - kT \ln \frac{N_{CB}}{N_D} \qquad E_F = E_{VB} + kT \ln \frac{N_{VB}}{N_A}$$

which can be written, based on the approximations derived as

$$E_F = E_{CB} - kT \ln \frac{N_{CB}}{n} \qquad E_F = E_{VB} + kT \ln \frac{N_{VB}}{p}$$

Carrier concentration for illumination:

$$n^*(x) = n + \Delta n$$

$$p^*(x) = p + \Delta p$$

$$E_{nF}^*(x) = E_{CB} - kT \ln \frac{N_{CB}}{n^*}$$

$$E_{pF}^*(x) = E_{VB} + kT \ln \frac{N_{VB}}{p^*}$$

$$E_{nF}^*(x) = E_{CB} - kT \ln\left[\frac{N_{CB}}{n^*}\right], \quad E_{pF}^*(x) = E_{VB} + kT \ln\left[\frac{N_{VB}}{p^*}\right].$$

For electron carriers

$$E_{nF}^*(x) = E_{CB} - kT \ln\left[\frac{N_{CB}}{n^*}\right],$$

and knowing

$$E_F = E_{CB} - kT \ln\left[\frac{N_{CB}}{n}\right] \quad \text{and} \quad E_{CB} = E_F + kT \ln\left[\frac{N_{CB}}{n}\right],$$

we can write

$$\begin{aligned} E_{nF}^*(x) &= E_F + kT \ln\left(\frac{N_{CB}}{n}\right) - kT \ln\left(\frac{N_{CB}}{n^*}\right) \\ E_{nF}^*(x) &= E_F + kT \left[\ln\left(\frac{N_{CB}}{n}\right) - \ln\left(\frac{N_{CB}}{n^*}\right) \right] \\ E_{nF}^*(x) &= E_F + kT \ln\left[\frac{N_{CB}/n}{N_{CB}/n^*}\right] \\ E_{nF}^*(x) &= E_F + kT \ln\left[\frac{n^*}{n}\right] \end{aligned}$$

Because $n^* = n + \Delta n$, we can write

$$E_{nF}^*(x) = E_{CB} + kT \ln\left[\frac{n + \Delta n}{n}\right] = E_{CB} + kT \ln\left[1 + \frac{\Delta n}{n}\right]$$

Quasi Fermi level for electron is energetically located above the dark Fermi level.

For hole carriers,

$$E_{pF}^*(x) = E_{VB} + kT \ln\left[\frac{N_{VB}}{p^*}\right].$$

Knowing

$$E_F = E_{VB} + kT \ln\left[\frac{N_{VB}}{p}\right] \quad \text{and} \quad E_{VB} = E_F - kT \ln\left[\frac{N_{VB}}{p}\right],$$

we can write

$$E_{pF}^*(x) = E_F - kT \ln\left(\frac{N_{VB}}{p}\right) + kT \ln\left(\frac{N_{VB}}{p^*}\right)$$

$$E_{pF}^*(x) = E_F - kT \left[\ln\left(\frac{N_{VB}}{p}\right) + kT \ln\left(\frac{N_{VB}}{p^*}\right) \right]$$

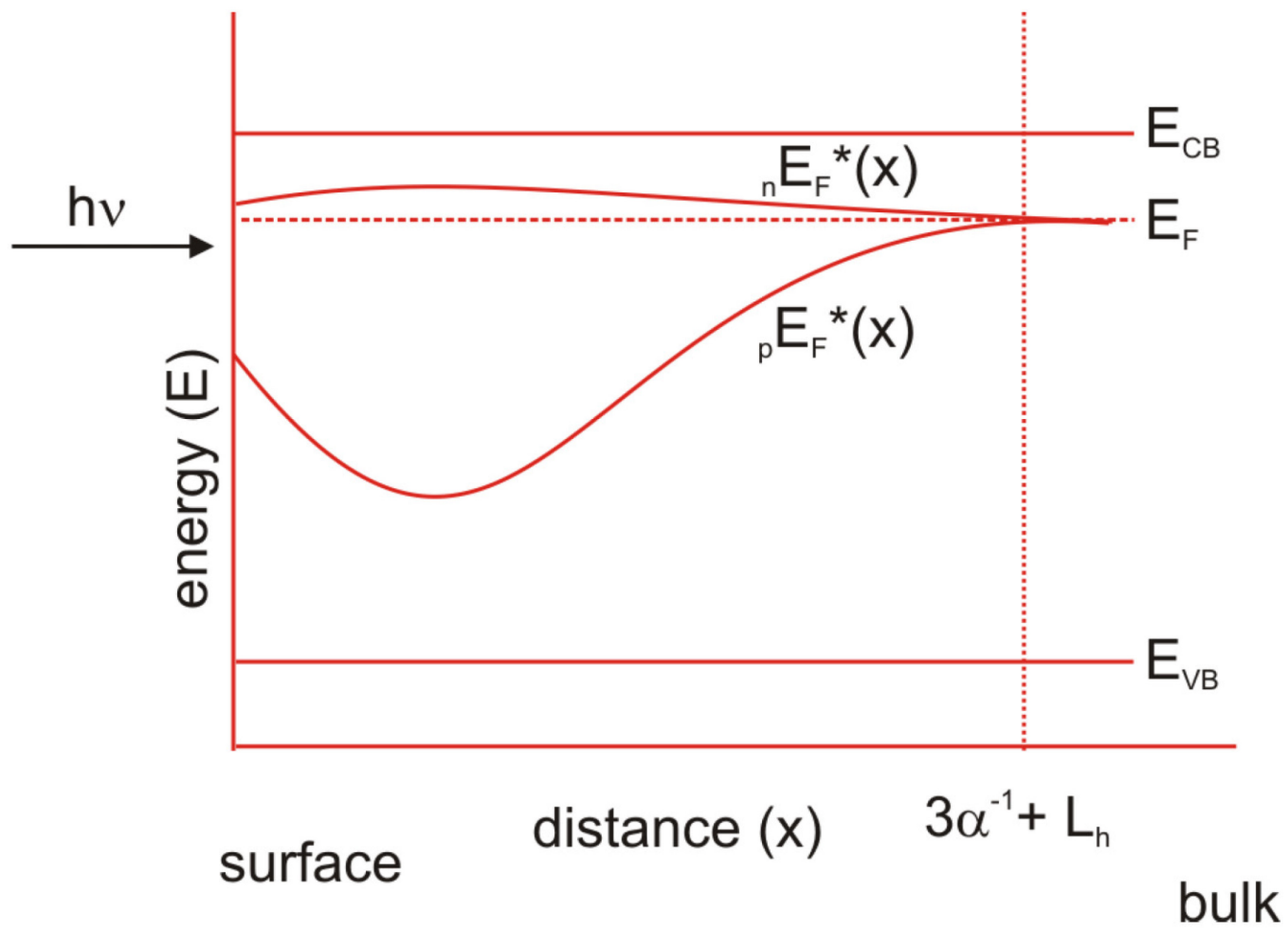
$$E_{pF}^*(x) = E_F - kT \ln\left[\frac{N_{VB}/p}{N_{VB}/p^*}\right]$$

$$E_{pF}^*(x) = E_F - kT \ln\left[\frac{p^*}{p}\right]$$

Because $p^* = p + \Delta p$, we can write

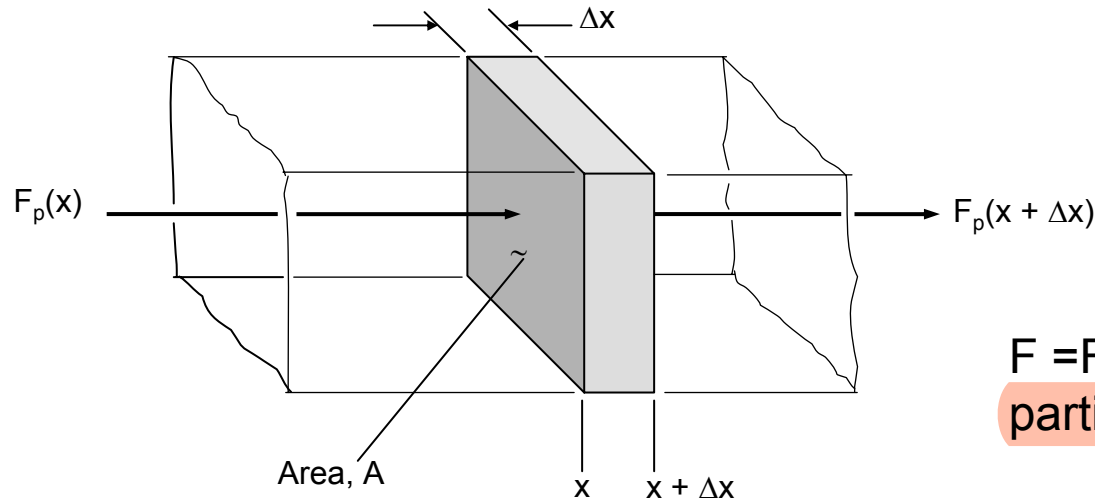
$$E_{pF}^*(x) = E_F - kT \ln\left[\frac{p + \Delta p}{p}\right] = E_F - kT \ln\left[1 + \frac{\Delta p}{p}\right]$$

Quasi Fermi level for hole is energetically located below the dark Fermi level.



Continuity equations:

The continuity equation describes the behavior of excess carriers with time and in space in the presence of electric fields and density gradients.



$F = F^+$ represents hole particle flux (# / cm² -s)

The net increase in hole concentration per unit time,

$$\left. \frac{\partial p}{\partial t} \right|_{x \rightarrow x + \Delta x} = \lim_{\Delta x \rightarrow 0} \frac{F_p^+(x) - F_p^+(x + \Delta x)}{\Delta x} + g_p - R_p$$

Continuity equations

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - R_p$$

The hole flux, $F_p^+ = J_p / e$ The unit of hole flux is holes/cm²-s

$$\frac{\partial p}{\partial t} = -\frac{1}{e} \frac{\partial J_p}{\partial x} + g_p - R_p$$

Similarly for electrons,

$$\frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial J_n}{\partial x} + g_n - R_n$$

These are the continuity equations for holes and electrons respectively

Recall: In one dimension, the electron and hole current densities due to the drift and diffusion are given by:

$$J_p = e\mu_p pE - eD_p \frac{\partial p}{\partial x} \qquad J_n = e\mu_n nE + eD_n \frac{\partial n}{\partial x}$$

Substitute these in the continuity equations:

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \frac{\partial(pE)}{\partial x} + g_p - R_p \qquad \frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + \mu_n \frac{\partial(nE)}{\partial x} + g_n - R_n$$

Or,

$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + \mu_n \left(E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - R_n$$

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \left(E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - R_p$$

$$n = n_0 + \delta n \quad \text{and} \quad p = p_0 + \delta p$$

The thermal-equilibrium concentrations, n_0 and p_0 , are not functions of time. For the special case of homogeneous semiconductor, n_0 and p_0 are also independent of the space coordinates. So the continuity may then be written in the form of:

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} + \mu_n \left(E \frac{\partial \delta n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - R_n$$

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p \left(E \frac{\partial \delta p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - R_p$$

Time –dependent
diffusion equations

Semiconductor Equations

The operation of most semiconductor devices can be described by the so-called semiconductor device equations, including

$$\nabla \cdot \varepsilon \vec{E} = q(p - n + N)$$

from Poisson's equation, where N is the net charge due to dopants and other trapped charges; and the hole and electron continuity equations

$$\nabla \cdot \vec{J}_p = q \left(G - R_p - \frac{\partial p}{\partial t} \right)$$

$$\nabla \cdot \vec{J}_n = q \left(R_n - G + \frac{\partial n}{\partial t} \right)$$

where G is the optical generation rate of electron-hole pairs. Thermal generation is included in R_p and R_n . The hole and electron current densities are given by

$$\vec{J}_p = -q\mu_p p \nabla(\phi - \phi_p) - kT\mu_p \nabla p$$

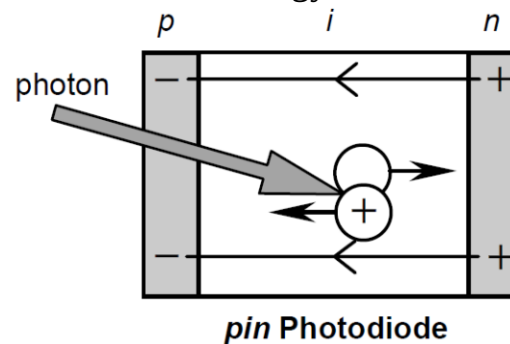
$$\vec{J}_n = -q\mu_n n \nabla(\phi + \phi_n) + kT\mu_n \nabla n.$$

The two terms, ϕ_p and ϕ_n , are the band parameters that account for degeneracy and a spatially varying band gap and electron affinity.

A complete description of the operation of optoelectronic devices can be obtained by solving the complete set of coupled differential equations Eq. (1), Eq. (2), and Eq. (3).

A case study of pin solar cell

(excerpted from the paper of E.A. Schiff, *Low-mobility solar cells: a device physics primer with application to amorphous silicon*, Solar Energy Materials & Solar Cells 78 (2003) 567–595)

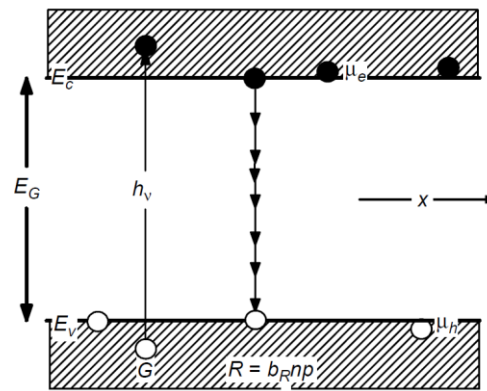


Analytical Model

The fundamental structure of the solar cells consists of three layers: a p-type electrode layer that collects holes, an intrinsic layer in which photocarriers are generated, and an n-type electrode layer to collect electrons.

The intrinsic layer is assumed to be an insulator under dark conditions. Excess electrons (holes) are donated from the n- (p-) type layer to the p- (n-) type layer, leaving these layers positively (negatively) charged, and creating a “built-in” electric field.

The photons are absorbed in the intrinsic layer, where each absorbed photon will generate one electron and one hole. The photocarriers are swept away by the built-in electric field to the n-type and p-type layers, respectively—thus generating solar electricity.



The electronic structure of the material is revealed with some critical parameters: the energy bandgap E_g , the effective densities of states $N_c(cm^{-3})$ and $N_v(cm^{-3})$ of the conduction and the valence bandedges, and effective mass $m_{e,h}$ and mobility $\mu_{e,h}$ of carriers in the conduction and valence bands.

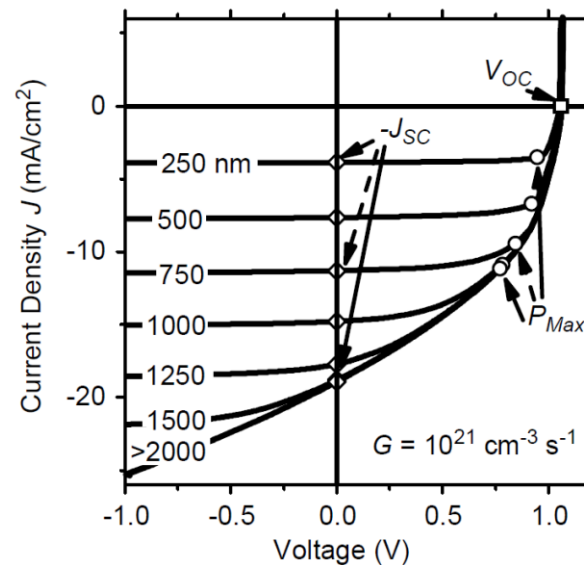
In the *i*-layer, photons with energies $h\nu$ (eV) greater than the bandgap $E_g = E_c - E_v$ can excite an electron out of the filled electronic levels in the valence band and into the conduction band; leaves one

hole behind in the valence band. The generation rate will be denoted by $G(\text{cm}^{-3} \cdot \text{s}^{-1})$.

The excess carriers rapidly shed energy and coalesce onto the levels close to the bandedges.

The average speed $v_{e,h} = \mu_{e,h} \cdot E$ of these carriers in an electric field E is characterized by the electron (hole) mobility $\mu_{e,h}(\text{cm}^2 / \text{V} \cdot \text{s})$.

The recombination rate $R(\text{cm}^{-3} \cdot \text{s}^{-1}) = b_R n p$ is proportional to the density of electrons n , the density of holes p , and the recombination coefficient $b_R(\text{cm}^3 \cdot \text{s}^{-1})$.

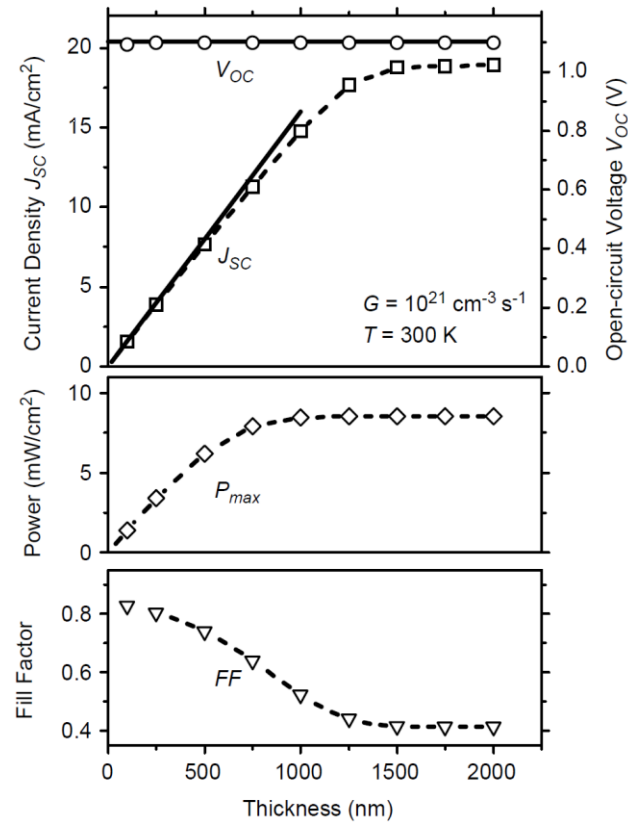


A positive voltage V corresponds to an external electric potential that is larger at the p -layer electrode than at the n -layer electrode. The value of V at which $J=0$ is called the “open-circuit” voltage V_{oc} .

At short-circuit ($V=0$), the magnitude of the current density is denoted the “short-circuit current density” J_{sc} .

No power is delivered to the external circuit under either short-circuit or open-circuit conditions. The power density $P(V) = J(V) \cdot V$ delivered to the external circuit is maximized at V_m (the maximum power point).

Under short-circuit conditions with a constant generation rate throughout the absorber layer, every photocarrier that is generated by photon absorption will be swept across the thin absorber layer by the internal electric field and collected.



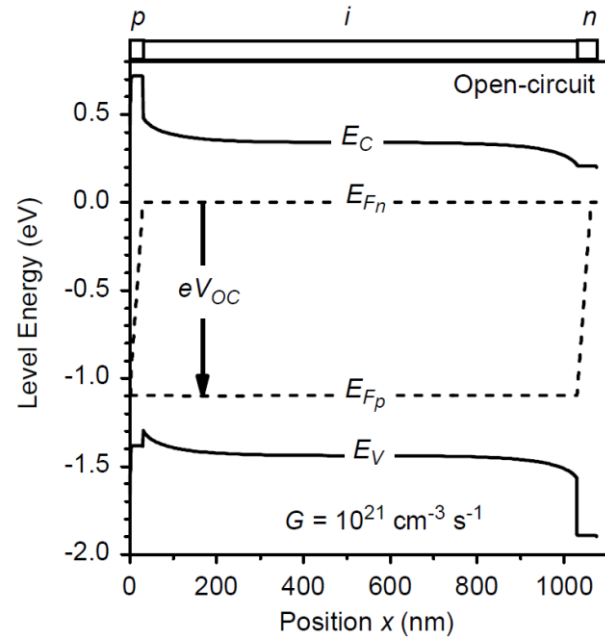
Therefore, $J_{sc} = eGd$. The short-circuit current density is proportional to thickness for $d < 1250 \text{ nm}$.

The open-circuit voltage V_{oc} is independent of thickness, and in terms of the fundamental parameters of the intrinsic layer material

$$eV_{OC} = E_g + kT \ln\left[\frac{G}{b_R N_c N_v}\right].$$

The fill-factor FF , which is defined from the power relation $P=(FF) \cdot J_{SC} \cdot V_{OC}$, saturates at about the same thickness as the short-circuit current density.

For intrinsic layers exceeding some collection width d_C (determined by the buildup of a space-charge of drifting carriers), the output power density P saturates at some maximum value. The power density saturates for smaller thicknesses than does the short-circuit current density or the fill-factor. This is due to that the electric potential across the cell is larger under short-circuit conditions than it is at the maximum power point. Photocarriers generated deeper in the cell than d_C may thus be collected under short-circuit conditions, but this additional collection does not contribute to power generation.



If the mobile electron density (i.e., the density of electrons occupying levels above the conduction bandedge E_c) is denoted by n , then the electron quasi-Fermi level E_{Fn} is defined by

$$E_{Fn} = E_c + kT \ln\left[\frac{n}{N_c}\right].$$

For holes, a similar expression can be defined

$$E_{Fp} = E_v - kT \ln\left[\frac{p}{N_v}\right].$$

Thus, photocarrier generation in a material creates two electronic reservoirs with differing chemical potentials. Electrons in the conduction band are one such reservoir; holes in the valence band are the second.

Electron and hole currents may be expressed in terms of quasi-Fermi energy gradient

$$J_n = n\mu_n E = n\mu_n \left(-\frac{\partial E_{Fn}(x)}{\partial x}\right).$$

Under open-circuit conditions, the intrinsic layer is in isolation (i.e., without gradients or electric fields that would transport electrons and holes in space). The n -layer serves as an ideal electrode, permitting us to measure E_{Fn} ; and similarly, the p -layer serves as an ideal electrode to measure E_{Fp} . A voltmeter connected to the n - and p -layers then measures $V_{OC} = (E_{Fn} - E_{Fp}) / e$.

Assuming the semiconductor layer in a device is electrically neutral ($n=p$). Under continuous illumination,

in steady state the rate of generation G is equal to the rate of recombination ($G = R = b_R np$), leading to the carrier densities $n = p = \sqrt{G/b_R}$. The quasi-Fermi energies of free carriers in the layer can be deduced to be

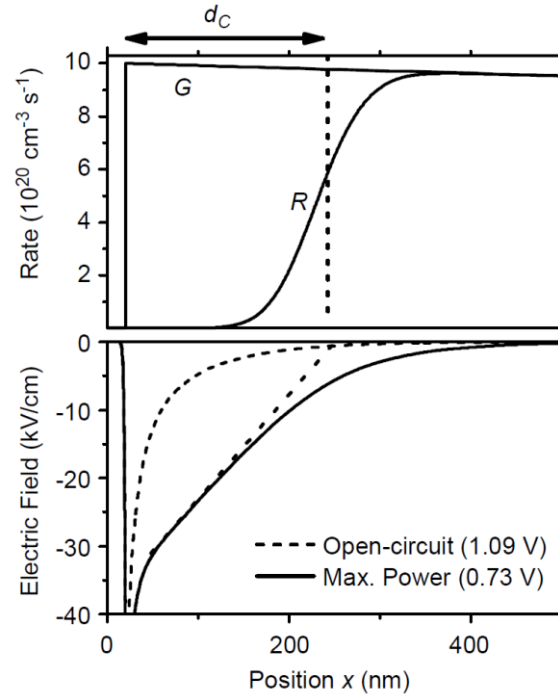
$$E_{Fn} = E_c + \frac{kT}{2} \ln\left[\frac{G}{b_R N_c^2}\right]$$

$$E_{Fp} = E_v - \frac{kT}{2} \ln\left[\frac{G}{b_R N_v^2}\right].$$

Thus the open-circuit voltage can be expressed as

$$eV_{OC} = E_g + kT \ln\left[\frac{G}{b_R N_c N_v}\right].$$

Under open-circuit conditions, the profile of the electric field (dashed curve) exhibits a quite large electric field near the p/i interface. Thus, hole photocarriers in this high field region drift rapidly towards the p -layer. However, this drift simply cancels the diffusion of holes out of the p -layer, resulting in a zero net current (drift-diffusion).



When the cell is biased at its maximum power point, the electric field penetrates more deeply into the cell, and net current is flowing. The field declines linearly towards zero at a depth d_C , identified as the collection width.

The generation rate profile $G(x)$ is essentially a constant. For $x > d_C$ deep in the device, all photocarriers that are generated recombine without being collected (i.e., $G=R$). However, every hole generated in d_C can

reach the p-layer and is collected. Thus, the current that flows in the external bias circuit may be estimated by $j = eGd_C$.

The electric field, which declines linearly to zero with depth, produces a uniform charge density ρ within collection zone. From semiconductor equations in the collection region ($0 < x < d_C$)

$$\begin{aligned}\frac{\partial J_h}{\partial x} &= e(G - R) \quad (\text{continuity equation}) \\ J_h(x) &= \rho(x)\mu_h E(x) \quad (\text{drift relation}) \\ \frac{\partial E(x)}{\partial x} &= \frac{\rho(x)}{\varepsilon} \quad (\text{Poisson's equation})\end{aligned} \quad .$$

At $x=0$, $R=0$ and $E(0) = -\rho d_C / \varepsilon$. From the continuity equation, $J_p(0) = -eGd_C$. From the drift relation,

$$J_p(0) = \rho\mu_p E(0) = -\rho^2\mu_p d_C / \varepsilon .$$

Thus, $\rho = \sqrt{e\varepsilon G / \mu_p}$. We note that $d_c = \sqrt{\Delta V} \cdot (4\mu_p \varepsilon / eG)^{1/4} = \sqrt{\Delta V} \cdot [4\varepsilon^2 / \rho^2]^{1/4}$ is determined by ρ and the electric potential $\Delta V = V(d_C) - V(0) = d_C^2 \rho / 2\varepsilon$ across the collection width.

The output power density is given by $P(V) = J(V) \cdot V$, in which $J(V)$ can be approximated with the hole

current $J_p(0) = eGd_C$ at $x=0$. $\Delta V = V_{OC} - V$ is the difference between the open-circuit voltage and the applied potential V . Thus, $P(V) \propto \sqrt{V_{OC} - V} \cdot V$, which reaches a maximum $P_m = (2V_{OC}/3)^{3/2}(\mu_p \varepsilon e^3 G^3)^{1/4}$ at $V_m = 2V_{OC}/3$. And $d_C = (2V_{OC}/3)^{1/2}(\mu_p \varepsilon / eG)^{1/4}$, implying that the maximum power density is a function of the hole mobility.

As the voltage across the cell is reduced from V_{OC} , photocarriers are collected in the region where the recombination rate R of electrons and holes falls below the generation rate G . The current density can be approximated with

$$j(V) = eGd(1 - \frac{R(V)}{G}),$$

where d is the thickness of the intrinsic layer. The applied potential $V(R)$ associated with a given

recombination rate R can be determined from $eV_{OC} = E_g + kT \ln[\frac{G}{b_R N_c N_v}]$ but replacing the generation rate G with the recombination rate, which yields

$$eV(R) \approx E_g + kT \ln\left[\frac{R}{b_R N_c N_v}\right]$$

for an arbitrary voltage V . Thus, we can deduce $e[V_{OC} - V(R)] = kT \ln\left[\frac{G}{R}\right]$, and

$J(V) = eGd[1 - \exp(-e\Delta V/kT)]$. The power density at an arbitrary voltage V becomes $P(V) = V \cdot J(V)$. A

maximum power density can be achieved with V_m , which shall fulfill the condition of

$$(e/kT)V_m = (e/kT)V_{OC} - \ln[1 + (e/kT)V_m].$$

For $b_R = 10^{-9} \text{ cm}^3 \text{ s}^{-1}$, we estimated $V_{OC} = 1.09 \text{ V}$. Solving the expression by iteration, we

obtain $V_m = 0.997 \text{ V}$, which is fairly close to the value $V_m = 0.97 \text{ V}$ for a 250-nm thick i -layer cell.

Effects of Valence Bandtail Width

Non-crystalline semiconductors do tend to have fairly low band mobilities. The best documented case is that of hydrogenated amorphous silicon, for which the electron band mobility is about $2 \text{ cm}^2/\text{Vs}$, compared to the value in crystal silicon ($1800 \text{ cm}^2/\text{V s}$ at 300 K). In practice, the effective mobility is further reduced by trapping of carriers into localized states between the bandedges.

An satisfactory account for drift-mobility measurements in amorphous a-Si:H could be given using an exponential distribution of these trap states. For the valence bandtail, the trap distribution vs. level energy is written

$$g_V(E) = g_V^0 (\text{cm}^3 / eV) \exp[-(E - E_V)/\Delta E_V].$$

ΔE_V is the width of the valence bandtail, which was found to be around 50 meV in a-Si:H. The conduction bandtail width is smaller around 22 meV, and near room-temperature ($kT=25 \text{ meV}$) conduction bandtail trapping can be neglected.

